An Electromagnetic Force Balance Experiment for Measurement Science Education

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Abstract. In the design and conception of all kinds of systems, measurement equipment and the respective measurement results play a crucial role. They provide the means to monitor the environment as well as the proper functioning of the respective systems. It is therefore mandatory to raise awareness of practitioners and engineers for important concepts such as measurement uncertainty, parameter uncertainty, their traceability and related consequences already in during education. In this paper, we present a teaching concept based on an electromagnetic force balance experiment towards the learning objectives such as electromagnetic force generation in actuators, utilization of the law of conservation of energy, experience with laboratory equipment and the SI system with aspects such as realizations of base units, traceability and in particular measurement uncertainty.

1. Introduction

In our modern society, measurement devices and sensors, the respective electronic evaluation and signal processing chains, are indispensable. Measurement science is employed to fulfil manifold tasks in consumer goods such as mobile phones and personal computers or tablets as well as in cars, (automated) production lines or safety and security systems. Virtually all electronic devices our society relies on in everyday private as well as working life are equipped with sensors and measurements electronics. When educating engineers to become the future designers of such measurement chains, critical analysis of sensor systems with respect to measurement uncertainty and traceability of results in context of standards, definitions ([1], [2]) and recent recommendations as well as research (e.g. [3] and [4]) thus seems mandatory. It is necessary to enhance the awareness with respect to measurement uncertainty, and uncertainty in modeling and parameter estimation in such systems and teach proper usage and adoption of the available methodology (- as in the Guide to the expression of Uncertainty in Measurement GUM [5, 6, 7] -) to prevent undesired system states, note at least in the context of safety critical systems. This is even more important when society more and more relies on electronic measurement systems, where in, e.g., automated production chains with human - robot interaction and collaboration, or autonomous in autonomous driving, lives depend on them.

In order to provide a practical experience in adoption, usage and especially limitations of the methods that are used to cope with these challenges, we provide a lab course for our Master students. Most of them are enrolled in the Master study program Information and Communications Engineering with a specialization in autonomous systems and robotics. It is assumed that the students are well familiar with the concept of measurement uncertainty
(compare [8, 9, 10]), probability theory, estimation and detection theory [11] as well as electrical measurements and sensors.

In the previous lectures and courses, the methodologies are presented theoretically or in simulation. The concept of this work is now to provide practical experience. We also aim at giving an understanding of the concepts behind the SI system and the definition of basic units as well as their traceability.

2. Electromagnetic Force Balance

Balances are well-known experimental setups, where the most established is the Watt balance (e.g. [12]). Other setups rely on electrostatic force (e.g. [13]) or electromagnetic force (e.g. [14]). The Watt balance is the experiment suggested to determine the Planck’s constant $h$ with uncertainties of $1$ part in $10^8$ and is also suggested for the realization of the kg [12]. It is developed to measure at the lowest feasible uncertainty, i.e. highest possible accuracy. Other mentioned force balances as presented in, e.g. [13] and [14], are designed for traceability and, as well, for high accuracy.

The principle of the experiment we consider, although the setup is by far not as accurate, is similar to those mentioned before. It relies on the principle of energy conservation and the balance of electromagnetic energy $E_{magn}$ and mechanical work $W_{mech}$ when no external energy is introduced. We can state the following relations

$$dW_{mech} = F ds$$  \hspace{1cm} (1)

$$E_{magn} = \frac{1}{2} LI^2$$  \hspace{1cm} (2)

$$dW_{mech} = -dE_{magn}$$  \hspace{1cm} (3)

$F$ is the force, $L$ is the inductance, $I$ is the current and $s$ is the path or distance. The $d$-prefix indicates changes in the respective quantities. We can further apply Faraday’s law of induction

$$U = N \frac{d\Phi}{dt} = L \frac{di}{dt}$$  \hspace{1cm} (4)

where $U$ is the voltage in a coil with $N$ windings, induced by a change in the magnetic flux $\Phi$. The current through the coil is denoted by $i$ for the dynamic (AC) and $I$ for the static (DC) quantity. Since the apparatus is used at equilibrium, i.e. no external energy is introduced, we know that voltage across the clamps needs to be zero (no electrical power), which further implies that the magnetic flux $\Phi$ needs to be constant, and we can thus modify (4) as

$$N\Phi = Li$$  \hspace{1cm} (5)

and reformulate as

$$i = \frac{N\Phi}{L}$$  \hspace{1cm} (6)

using (1) and (2) and the assumption of constant magnetic flux $\Phi$, we get

$$F = \frac{dW_{mech}}{ds} = -\frac{dE_{magn}}{ds} \bigg|_{\Phi=const} = -\frac{1}{2L^2} (N\Phi)^2 \frac{dL}{ds} = -\frac{1}{2} \left( \frac{N\Phi}{L} \right)^2 \frac{dL}{ds}$$  \hspace{1cm} (7)

and using (6) we find

$$F = -\frac{1}{2} I^2 \frac{dL}{ds}$$  \hspace{1cm} (8)
3. Measurements and Evaluations

Using the setup as shown in Fig. 1, the coil can be moved above the ferromagnetic metal piece placed on the scale. The gradient of the inductance with respect to the path (distance between coil and ferromagnetic metal piece), i.e. \( \frac{dL}{ds} \), is measured (either with an LCR bridge or scope/multimeter and signal generator). The distance can be measured either with the ruler mounted at the height adjustment facility or using a caliper. The force is to be determined using the Eq.s as given above and is to be compared with the reference measurements of the electronic scale (see Fig. 1). To determine the inductance or change of inductance of the coil using the multimeter, it is necessary to record the change of the current \( i \) at fixed voltage \( u \) and frequency \( f \). Using these values, it is possible to determine the coil inductance via impedance, using

\[
Z = \frac{u}{i} = \sqrt{R^2 + X_L^2}
\]

(9)

where the complex inductive resistance is

\[
X_L = j\omega L
\]

(11)

and where \( \omega \) is the angular frequency and can be rewritten as \( \omega = 2\pi f \). The inductance \( L \) can then be calculated e.g. as

\[
L = \frac{\sqrt{Z^2 - R^2}}{j\omega}
\]

(12)

The measured current \( i \) as function of the distance \( d \) at a constant voltage is given in Fig. 2 in the upper left. The respective inductance value is plotted in Fig. 2 upper right. The force as function of the current at a fixed distance is illustrated in Fig. 2 lower right. Bringing conductive material close to an energized coil, we can observe two different phenomena:

(i) In conductive material which is not magnetic (copper, aluminum etc.), the magnetic field of the approaching coil causes eddy currents. These eddy currents then produce a magnetic field which counteracts the primary field of the coil. This effect is used in, e.g., proximity sensors or non destructive testing of metallic parts.
(ii) For ferromagnetic material, the prevalent effect is an increase in inductance due to the increased permeability $\mu_r$ (magnetic material instead of air). The magnetic field is enforced through the ferromagnetic material and its magnetization. This principle is used in transformers with ferromagnetic cores. The generation of eddy currents occurs in ferromagnetic material as well. To avoid this unwanted effect in, e.g., transformer cores, they are commonly made of electrically isolated sheet plates.

The lower left plot in Fig. 2 gives the force as dependent on the distance, once measured using the scale (blue) and once derived via inductance (black). Obviously, the curve for the derived force is noisier than the others. Here we see uncertainties introduced from the used measurement devices (e.g., resolution, accuracy, settling time) as well as a low number of measurement points. Two types of fits are determined to describe the derived force as dependent on the distance using a semi-analytic model: standard and robust. Here, the choice of suitable model is crucial, in this case, we use a model of the form

$$F = \beta_1 e^{-\frac{1}{\beta_2 d^2}} + \frac{1}{\beta_3 d}$$  \hspace{1cm} (13)
where the vector $\beta$ holds the model coefficients determined through the fitting routine. Using the standard fit, the model coefficients are determined using a simple least squares approach

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

where $\hat{\beta}$ are the estimated coefficients, $X$ is the measurement matrix, and $y$ is the response. Often, these measured quantities used to determine the response are also called predictors. In the weighted least squares, points are given more weights which are close to the fitted line, thus the influence of outliers (and large variation in measurement) can be reduced

$$\hat{\beta} = (X^T W X)^{-1} X^T W y$$

where $W$ is the weight matrix. In the case of the derived force as dependent on the distance, this robust fit can reduce the discrepancy between measured and derived dependence. Other approaches include the determination of outliers (e.g. violating the $1.5\sigma$ bound) and remove them before fitting.

4. Traceability
Traceability is defined according to the VIM [2] as "A property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty."

Traceability, as the name already implies, defines a concept which enables the experimenter to state the measurement uncertainty as dependent on the used equipment and setup. To be able to do so, it is necessary that all the used equipment is calibrated and the remaining uncertainty in the measurement is stated. The experimental setup used in this work is not traceable: the measurement equipment (ruler or caliper) of the distance between coil and ferromagnetic metal piece is not unique: using the ruler, there is no calibration routine performed, using the caliper no unique measuring position as well as procedure has been defined. The same is the case for the height adjustment facility.

5. General Teaching Concept
The respective course where this concept is applied is a laboratory course under the title of sensors and actuators. Since this course is suggested as successor to the Measurement and Signal Processing (MSP) course. In MSP, advanced stochastic concepts are covered to yield, in a certain sense, optimal estimators for measurements subject to various sources of noise. Thus, students are assumed familiar with probability theory and statistics. Hence, mathematical background to deal with measurement uncertainty is assumed known, as well as parameter estimation principles and regression.

The students are advised to form groups. In each unit, a group performs one of the four specifically designed experiments. Each experiment is elaborated to gain experience with laboratory equipment as well as the respective physics and theory.

In the considered experimental design, the students are supposed to

- Measure inductance as a function of the distance between a coil and a ferromagnetic part with signal generator and scope/multimeter and with LCR-bridge
- Perform regression: get a fitted analytic function for the inductance as function of the distance
- Determine the force as a function of the current
- Validate the results using the electronic scale
Perform an uncertainty evaluation: is our result for the force traceable?

The preliminary theory and equipment basics are introduced and explained at the beginning of each unit. At the end, the experimentally achieved results are discussed and have to be gathered in a report. Additionally, students are encouraged to give suggestions for improvement of the considered setup.

6. Suggestions for Improvement
Suggestions for improvement are given in terms of the accuracy as well as resolution. Contributors such as measurement instruments, environmental conditions and the measurement process are addressed. A few results are gathered in the following:

- The measurement of inductance using the LCR-meter is preferred over the calculation via impedance.
- Improved mounting of the coil (slack and instable)
- Height adjustment resolution is too small
- Place ruler (for distance measurement) differently: e.g., closer to the coil

7. Conclusion
The presented experiment is a simple to realize laboratory setup, based on which students can be familiarized with basic principles of sensors and actuators, measurement uncertainty, system modelling, the SI system and traceability. The experiment allows to demonstrate the influence of the uncertainties of the contributing equipment on the system. The principles of curve fitting and modelling are demonstrated by determining a semi-analytical model for dependence of the change of inductance \( dL \) on the change of distance \( ds \). Furthermore, the difficulty in finding a suitable fit and influence of uncertainty in measurement are demonstrated.

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References
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